

Vibrations influence on the operation of gears

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Abstract. The purpose to optimize gears requires a very good understanding of their dynamic behaviour. Very important is the influence of vibrations on the operation of the gears. As a result of environmental constraints such as acoustics, gears are increasingly subject to stringent requirements in terms of load capacity, efficiency, noise and vibrations. The deformations of the gears involve deformations, in particular, located near the contacts that require great finesse. Gearboxes are equally sources of current vibrations as they are commonly used in industry. They are the subject of numerous studies to characterize their kinematic and dynamic behaviour. At present, research activities focus on the development of multidisciplinary experimental, theoretical and numerical skills that are put into play when designing structures, machine elements or, in general, mechanical systems. In this paper, we have proposed to find the dynamic response of the gearbox to the action of external disturbing factors. To this end, we have called for the use of integral transformations which, algebraizing the problem, greatly simplifies calculations.

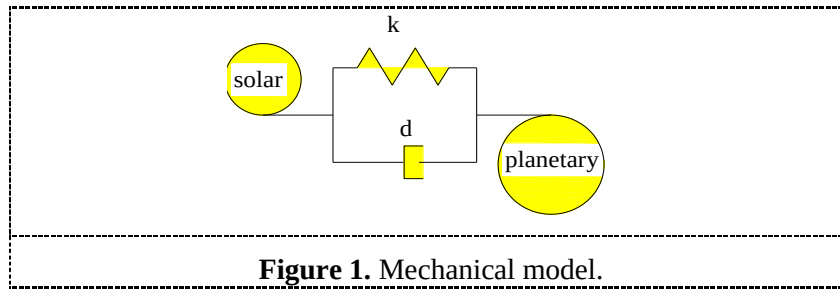
1. Introduction

The control and estimation of vibrations of mechanical systems is an important part of the design of new products [1]. Numerous sources of vibration may exist in a system such as friction and shocks [2, 3, 4, 5]. Gearboxes are equally sources of current vibrations as they are commonly used in industry. They are the subject of numerous studies to characterize their cinematic and dynamic behavior [6, 7, 8]. There are various approaches to modeling vibrations and gear noise [9, 10, 11]. The literature highlights the need to integrate both the elasticity of gear components and the descriptions of its vibrations in a three-dimensional dynamic behavior model [12, 13]. The deformations of the gears involve deformations, in particular, located near the contacts that require great finesse. As a result, we are encouraged to move towards a simplified model with respect to the three-dimensional finite element models, yet still sufficiently precise to study the elasticity of the engaging elements while permitting the integration of relatively complex geometries of the deformable wheels [14, 15]. As we have already shown, engagement appears to be the main source of excitation in power transmission components. The instantaneous movements of each wheel are represented by six degrees of freedom (three translations and three rotations). In [16] there is presented, in a multidisciplinary approach, the simplified mechanical model for the vibro-acoustic study of a gear, the elements of the gearing behaving like rigid solids. The objective of this approach is to optimize a mechatronic product. At present, research activities focus on the development of multidisciplinary experimental, theoretical and numerical skills that are put into play when designing structures, machine elements or, in general, mechanical systems [17, 18].

The aim of the research is to improve knowledge of the behavior of materials and structures, develop models and tools useful in designing structures and machines, and harness a technical culture of analytical, designing and manufacturing methodologies. This research is based on materials science, nonlinear mechanics of solids, fluid and coupled systems, acoustics, training and processing techniques, experimental measurement methods and numerical modeling. The appearance of simulation times is the capital for this type of analysis where the number of iterations can be significant. For this reason, it was decided to model the reducer by approaching rigid bodies. At the same time, the need to obtain vibrations on the outer surface of the gear requires the finite element method to be applied to the latter. The first part of the article shows the modeling obtained by coupling multiple rigid bodies with the finite elements of an epicycloidal reducer. The second part illustrates the multi physical modeling with Dymola. It is here to model an epicycloids reducer with straight teeth, proposing for this purpose a hybrid model of multiple rigid bodies / finite elements [19, 20]. A more complete hybrid model is found in the paper [21]. In this paper, we have proposed to find the dynamic response of the gearbox to the action of external disturbing factors. To this end, we have called for the use of integral transformations that, algebraizing the problem, greatly simplifies calculations.

2. Mechanical and mathematical models

In Figure 1, simulation of gearing is managed through the mechanical model that contains a spring and a shock absorber in parallel.



The mathematical model of the movement could be written in matrix form:

$$M\ddot{q} + D\dot{q} + Kq = F \quad (1)$$

The matrix of masses M , the damping matrix D , the stiffness matrix K and the excitation matrix F having the following representations, respectively:

$$M = \begin{bmatrix} M_s & 0 & 0 & 0 & 0 \\ 0 & M_{p1} & 0 & 0 & 0 \\ 0 & 0 & M_{p2} & 0 & 0 \\ 0 & 0 & 0 & M_{p3} & 0 \\ 0 & 0 & 0 & 0 & M_{p4} \end{bmatrix} \quad (2)$$

$$D = \begin{bmatrix} d_{sp} & -d_{sp1} & -d_{sp2} & -d_{sp3} & -d_{sp} \\ 0 & d_{sp1} + d_{rp1} & 0 & 0 & d_{sp1} - d_{rp1} \\ 0 & 0 & d_{sp2} + d_{rp2} & 0 & d_{sp2} - d_{rp2} \\ 0 & 0 & 0 & d_{sp3} + d_{rp3} & d_{sp3} - d_{rp3} \\ 0 & 0 & 0 & 0 & d_{sp} + d_{rp} \end{bmatrix} \quad (3)$$

$$K = \begin{bmatrix} k_{sp} & -k_{sp1} & -k_{sp2} & -k_{sp3} & -k_{sp} \\ 0 & k_{sp1} + k_{rp1} & 0 & 0 & k_{sp1} - k_{rp1} \\ 0 & 0 & k_{sp2} + k_{rp2} & 0 & k_{sp2} - k_{rp2} \\ 0 & 0 & 0 & k_{sp3} + k_{rp3} & k_{sp3} - k_{rp3} \\ 0 & 0 & 0 & 0 & k_{sp} + k_{rp} \end{bmatrix}; F = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \end{bmatrix}. \quad (4)$$

In which:

$$k_{sp} = \sum_{i=1}^3 k_{spi}, d_{sp} = \sum_{i=1}^3 d_{spi}, k_{rp} = \sum_{i=1}^3 k_{rpi}, d_{rp} = \sum_{i=1}^3 d_{rpi} \quad (5)$$

Is defined also the angle $\psi_i = \phi_i - \alpha$.

The meaning of the indices is:

- ✓ $s, p1, p2, p3$ respectively represent the sun, the three planets and the satellite port;
- ✓ spi represents the contact between solar and planetary i ;
- ✓ rpi represents the contact between the crown and the planetary i ;
- ✓ the angle ϕ_i positions the center of the planet and the associated satellite port map;
- ✓ α is the pressure angle.

3. Dynamic response

We consider the system not amortized:

$$M \ddot{q} + Kq = F \quad (6)$$

Applying the transformed Laplace linear system (6) under homogeneous initial conditions:

$$q_i(0) = 0, \dot{q}_i(0) = 0, i = \overline{1,5} \quad (7)$$

Results in an algebraic system, which has as unknown the Laplace coordinates $q_i(s)$, $i = \overline{1,5}$ of the coordinates $q_i(t)$, $i = \overline{1,5}$. By basically solving this system, we get the Laplace coordinates $q_i(s)$, $i = \overline{1,5}$ of the coordinates $q_i(t)$, $i = \overline{1,5}$, as follows:

$$q_1(s) = \left[F_1 - \frac{b}{M_{p1}s^2 + f} \left(F_2 - \frac{gF_5}{M_{p4}s^2 + 1} \right) - \frac{c}{M_{p2}s^2 + h} \left(F_3 - \frac{iF_5}{M_{p4}s^2 + 1} \right) - \frac{d}{M_{p3}s^2 + j} \left(F_4 - \frac{kF_5}{M_{p4}s^2 + 1} \right) - \frac{eF_5}{M_{p4}s^2 + 1} \right] \bullet \frac{1}{M_s s^2 + a}; \quad (8)$$

$$q_2(s) = \frac{1}{M_{p1}s^2 + f} \left(F_2 - \frac{gF_5}{M_{p4}s^2 + 1} \right); q_3(s) = \frac{1}{M_{p2}s^2 + h} \left(F_3 - \frac{iF_5}{M_{p4}s^2 + 1} \right); \quad (9)$$

$$q_4(s) = \frac{1}{M_{p3}s^2 + j} \left(F_4 - \frac{kF_5}{M_{p4}s^2 + l} \right); \quad q_5(s) = \frac{F_5}{M_{p4}s^2 + l} \quad (10)$$

Reversing Laplace transformations of Lagrange coordinates, we obtain below the time functions $q_i(t)$, $i = \overline{1, 5}$, which represent the dynamic response of the gear unit to disturbing external stresses.

$$\begin{aligned} q_1(t) = & \frac{F_1}{\sqrt{M_s a}} \sin \left(\sqrt{\frac{a}{M_s}} t \right) - \frac{bF_2}{\sqrt{M_{p1} f}} \sin \left(\sqrt{\frac{f}{M_{p1}}} t \right) + \\ & + \frac{bgF_5}{lM_{p1} - M_{p4}f} \left[\sqrt{\frac{M_{p1}}{f}} \sin \left(\sqrt{\frac{f}{M_{p1}}} t \right) - \sqrt{\frac{M_{p4}}{l}} \sin \left(\sqrt{\frac{l}{M_{p4}}} t \right) \right] - \frac{cF_3}{\sqrt{M_{p2} h}} \sin \left(\sqrt{\frac{h}{M_{p2}}} t \right) + \\ & + \frac{ciF_5}{lM_{p2} - M_{p4}h} \left[\sqrt{\frac{M_{p2}}{h}} \sin \left(\sqrt{\frac{h}{M_{p2}}} t \right) - \sqrt{\frac{M_{p4}}{l}} \sin \left(\sqrt{\frac{l}{M_{p4}}} t \right) \right] - \frac{dF_4}{\sqrt{M_{p3} j}} \sin \left(\sqrt{\frac{j}{M_{p3}}} t \right) + \\ & + \frac{dkF_5}{lM_{p3} - M_{p4}j} \left[\sqrt{\frac{M_{p3}}{j}} \sin \left(\sqrt{\frac{j}{M_{p3}}} t \right) - \sqrt{\frac{M_{p4}}{l}} \sin \left(\sqrt{\frac{l}{M_{p4}}} t \right) \right] - \\ & - \frac{eF_5}{aM_{p4} - M_s l} \left[\sqrt{\frac{M_{p4}}{l}} \sin \left(\sqrt{\frac{l}{M_{p4}}} t \right) - \sqrt{\frac{M_s}{a}} \sin \left(\sqrt{\frac{a}{M_s}} t \right) \right]; \end{aligned} \quad (11)$$

$$\begin{aligned} q_2(t) = & \frac{F_2}{\sqrt{lM_{p4}}} \sin \left(\sqrt{\frac{l}{M_{p4}}} t \right) - \\ & - \frac{gF_5}{lM_{p1} - M_{p4}f} \left[\sqrt{\frac{M_{p1}}{f}} \sin \left(\sqrt{\frac{f}{M_{p1}}} t \right) - \sqrt{\frac{M_{p4}}{l}} \sin \left(\sqrt{\frac{l}{M_{p4}}} t \right) \right]; \end{aligned} \quad (12)$$

$$\begin{aligned} q_3(t) = & \frac{F_3}{\sqrt{hM_{p2}}} \sin \left(\sqrt{\frac{h}{M_{p2}}} t \right) - \\ & - \frac{iF_5}{hM_{p1} - M_{p2}l} \left[\sqrt{\frac{M_{p4}}{l}} \sin \left(\sqrt{\frac{l}{M_{p4}}} t \right) - \sqrt{\frac{M_{p2}}{h}} \sin \left(\sqrt{\frac{h}{M_{p2}}} t \right) \right]; \end{aligned} \quad (13)$$

$$\begin{aligned} q_4(t) = & \frac{F_4}{\sqrt{jM_{p3}}} \sin \left(\sqrt{\frac{j}{M_{p3}}} t \right) - \\ & - \frac{kF_5}{jM_{p4} - M_{p3}l} \left[\sqrt{\frac{M_{p4}}{l}} \sin \left(\sqrt{\frac{l}{M_{p4}}} t \right) - \sqrt{\frac{M_{p3}}{j}} \sin \left(\sqrt{\frac{j}{M_{p3}}} t \right) \right]; \end{aligned} \quad (14)$$

$$q_5(t) = \frac{F_5}{\sqrt{IM_{p4}}} \sin\left(\sqrt{\frac{1}{M_{p4}}} t\right) \quad (15)$$

Where:

$$\begin{aligned} a &= k_{sp}; b = -k_{sp1}; c = -k_{sp2}; d = -k_{sp3}; e = -k_{sp}; f = k_{sp1} + k_{rp1}; g = k_{sp1} - k_{rp1}; \\ h &= k_{sp2} + k_{rp2}; i = k_{sp2} - k_{sp1}; j = k_{sp3} + k_{rp3}; k = k_{sp3} - k_{rp3}; l = k_{sp} + k_{rp} \end{aligned} \quad (16)$$

4. Conclusions

Lately, as we know, the interest in using computers has increased considerably, including in the area of their application to various transformations and symbolic calculations. This is due, among other things, to the development of scheduled means that reduce the need to develop its own programs to solve various problems. Electronic analytical calculation systems, such as Mathematica, MatLab, Maple, or Mathcad, include thousands of commands included and library functions as well as visualization of the plotting process. Applying the unilateral Laplace transform in relation to time had the purpose in this paper to highlight its role in simplifying integrative operations, if we compare it with traditional methods, the role of mathematics being essential in the studies made by the researcher. Thus, it is unlikely that there could be serious scientific projects that do not have mathematical relevance and were completed without the use of electronic computers. A high level of security of calculations can ensure the use of several systems of computational mathematics. Many of the computational methods used are based on matrix formalism, easy to implement on the computer. Much of the solutions of differential equations, which describe, for example, the behavior of the material systems at the action of the internal and external factors acting on them, can be obtained with the help of the computer. To validate mechanical models an important role surely has dynamic tests. Thus, an important role is played by the modal experimental analysis problems, as well as the way in which the calculation-test correspondence can be done, as well as the updating of the model. Often used in mechanics and, in particular, in vibration mechanics are inverse problems. Among the applications and methods approached in this respect we can cite the inverse problems of the sources, the optimization of the shape, the active control of the fluid transport devices or, finally, the methods of the integral equations which are widely used in mechanics and acoustics. Such a research project can be conceived only in relation to other scientific concerns of the researcher or scientist, requiring the development of a series of competences such as:

- ✓ determination of the dynamic behaviour of complex mechanical systems in the design phase;
- ✓ development and evaluation of fast and solid techniques for the study of the stability of delayed systems (time simulation, multi-frequency methods, root clustering techniques);
- ✓ modelling of specific phenomena (cutting of metals) and interactions in the system in the presence of vibrations;
- ✓ development of techniques for active control of vibration phenomena in mechanical processing (delay control systems);
- ✓ taking into account the dynamic constraints in the design phase of the machines;
- ✓ integrating and managing models on different scales.

At present, research activities focus on the development of multidisciplinary experimental, theoretical and numerical skills that are put into play when designing structures, machine elements or, in general, mechanical systems. The aim of the research is to improve knowledge of the behaviour of materials and structures, develop models and tools useful in designing structures and machines, and harness a technical culture of analytical, designing and manufacturing methodologies.

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